Definition of Motion (M4NA Declaration):

"Motion is the dynamic process for an object traveling through a distance over time, determined by its velocity and the cause of an applied force".

Mathematical Definition of M4NA Motion:

"Motion is characterized by velocity which is the actual traveling distance of an object per unit time in a specific direction".

For a particular period of actual travel: $\boxed{v = \frac{s}{t}}$

where:

- s= **actual traveling distance**
- t= time taken
- v= velocity in the direction of that travel

So equivalently: $\boxed{s = vt}$

Actual traveling distance

Suppose an object travels with different velocities during successive time intervals: $v_1, v_2, v_3, \dots, v_n$ for corresponding times: $t_1, t_2, t_3, \dots, t_n$. The distance travelled during each interval is: $s_1 = v_1 t_1, s_2 = v_2 t_2, \ s_3 = v_3 t_3, \dots., \ s_n = v_n t_n$.

Therefore, total actual traveling distance is the accumulation: $\boxed{S = s_1 + s_2 + s_3 + \cdots + s_n}$
Substituting each $s_i = v_i t_i$: $\boxed{S = v_1 t_1 + v_2 t_2 + v_3 t_3 + \cdots + v_n t_n}$ or compactly:
$\boxed{S = \sum_{i=1}^{n} v_i \, t_i}$ This is what we mean by **time-weighted accumulation of velocity**.

Total elapsed time

The total time is: $\boxed{T = t_1 + t_2 + t_3 + \cdots + t_n}$ or: $\boxed{T = \sum_{i=1}^{n} t_i}$

Average velocity

Average velocity is the total actual traveling distance divided by total elapsed time: $\boxed{\bar{v} = \frac{S}{T}}$

Now substitute the accumulated distance: $\bar{v} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \cdots + v_n t_n}{t_1 + t_2 + t_3 + \cdots + t_n}$

Therefore: $\boxed{\bar{v} = \frac{\sum_{i=1}^{n} v_i t_i}{\sum_{i=1}^{n} t_i}}$

Why is it called time-weighted?

Because each velocity is multiplied by the amount of time it operates: $v_1\boxed{t_1}$, $v_2\boxed{t_2}$, $v_3\boxed{t_3}$, ...

A velocity that operates for a longer time contributes more to the total traveling distance.

If all time intervals are equal

Suppose: $t_1 = t_2 = t_3 = \cdots = t_n = t$.

Then: $S = t(v_1 + v_2 + v_3 + \cdots + v_n)$ and: $T = nt$.

Therefore: $\bar{v} = \frac{t(v_1+v_2+\cdots+v_n)}{nt}$.

The t's cancel: $\boxed{\bar{v} = \frac{v_1+v_2+\cdots+v_n}{n}}$

So, the ordinary arithmetic mean is only obtained when the velocities are measured over **equal time intervals**.

Continuous form

When the time intervals become infinitesimally small: $\boxed{S = \int_{t_0}^{t_1} v\,(t)\,dt}$

and therefore: $\boxed{\bar{v} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} v\,(t)\,dt}$

So, the complete M4NA mathematical sequence we have been discussing can be written: $\boxed{v_i t_i \rightarrow \sum v_i\, t_i \rightarrow S \rightarrow \frac{S}{T} = \bar{v}}$ or, in expanded form: $\boxed{\bar{v} = \frac{v_1 t_1 + v_2 t_2 + \cdots + v_n t_n}{t_1 + t_2 + \cdots + t_n}}$

This is the **detailed mathematical form of the time-weighted velocity accumulation**.

Thus: $\boxed{S = \sum_{i=1}^{n} v_i\, t_i}$ and the **average velocity** over the whole period is: $\boxed{\bar{v} = \frac{\sum_{i=1}^{n} v_i t_i}{\sum_{i=1}^{n} t_i}}$

Therefore, distinguish the two

Velocity at a particular interval: $\boxed{v_i = \frac{s_i}{t_i}}$

Average velocity over the whole journey: $\boxed{\bar{v} = \frac{s_1+s_2+\cdots+s_n}{t_1+t_2+\cdots+t_n}}$

Since $s_i = v_i t_i$: $\boxed{\bar{v} = \frac{v_1 t_1+v_2 t_2+\cdots+v_n t_n}{t_1+t_2+\cdots+t_n}}$

Types of M4NA Motion:

1. Uniform Motion and
2. Non-uniform motion:
   a) Accelerative Motion and
   b) Non-Accelerative Motion:
      i) Deceleration,
      ii) Acceleration Deceleration and
      iii) Others